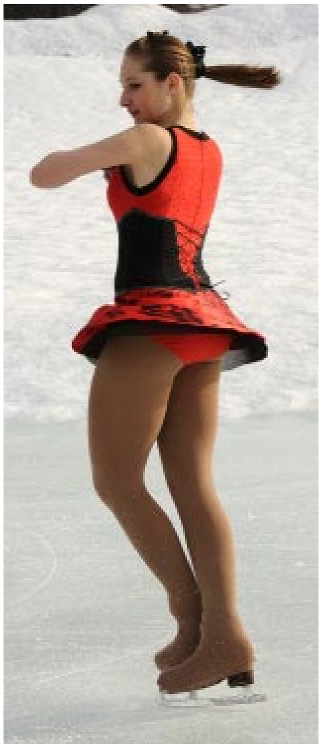
1. Conservation of Angular Momentum

An object that is spinning, or has internal spinning components, has a quantity associated with it known as its angular momentum. This momentum is expressed as the vector sum of the inertia of each spinning component times the rotational rate of that same component. Note that this is a vector sum because, in general, the inertia is actually a 3x3 *tensor*, comprised of 9 elements (6 independent) and the rotational rate is a 3x1 vector (comprised of three orthogonal rate components). Mathematically, the angular momentum of an object can be expressed as:

$$h = \sum_i I_i \omega_i$$

where h is the 3x1 angular momentum vector, I_i is the inertia tensor of the i^{th} spinning mass element and ω_i is the rotational rate vector of the i^{th} spinning mass element (known also as Euler rates). Most often, SI (System International) units are used to express angular momentum. If SI units are used, the inertia tensor must be expressed in kilograms*meters^2 and the Euler rates must be expressed in radians per second. The resulting angular momentum has the units of Newtons*meters*seconds (recall that a Newton is really one kilogram*meter per second^2).

With a good understanding now of what angular momentum is, one can now imagine how the total angular momentum, h, of an object can remain constant. Take, for example, a spinning figure skater.

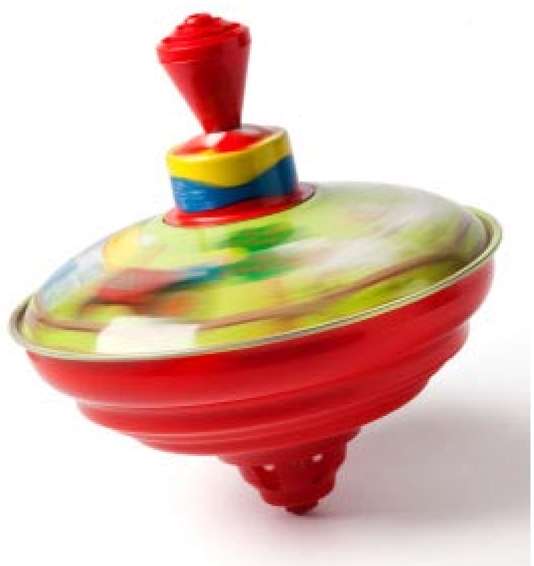


Considering only motion about the axis of the athlete (from her head to her toes), the (now scalar) angular momentum can be written as:

$$h = I\omega$$

The concept of conservation of angular momentum is only true in the absence of any external torque. Under the assumption that the figure skater is spinning on the ice with no torques applied by her skates, how is it then that a skater can change her rotational speed? The answer lies in the conservation of angular momentum. By extending her arms, she increases her inertia. By examining the equation above, if I increases, and h is to remain constant, then ω must decrease. Conversely, if the skater were to reduce her inertia by bringing her arms closer to her torso, her rotational speed must increase.

The figure skater scenario illustrates a single-axis (scalar) example of the conservation of angular momentum. However, since angular momentum is actually a vector quantity, a vector example is also presented. Consider a spinning top:



The angular momentum (now a 3x1 vector) must remain constant under the assumption that there are no external torques. This means that not only must the speed and inertia remain constant, but the axis of rotation must also remain constant. This is why a spinning top feels “stiff” when pushed on lightly. This stiffness is known as “gyric stiffness”, which will be described in more detail later.